New Constructions of MACs from Tweakable Block Ciphers

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Outline

A quick overview of our results

We propose two Nonce-based, two Randomized, and two Deterministic MAC constructions based on a $\varepsilon-$AXU and uniform hash function and a Block Cipher or a Tweakable Block Cipher which are:

- efficient (1 call to the underlying cipher and 1 or 2 calls to the hash function),
- provably (very) secure, in the Ideal Cipher model for BC-based constructions and in the Standard Model for TBC-based ones.

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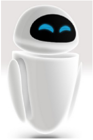
Nonce-Based Message Authentication Codes

(N, M, T) →



$T = \mathrm{MAC}_K(N, M)$

$\mathrm{MAC}_K(N, M) = T\ ?$

Security Definition

The adversary is allowed

- q_m MAC queries $T = \mathrm{MAC}_K(N, M)$
- q_v verification queries (forgery attempts) (N', M', T')

and is successful if one of the verification queries (N', M', T') passes and no previous MAC query (N', M') returned T'.

The adversary is said nonce-respecting if it does not repeat nonces in MAC queries.

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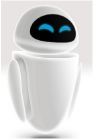

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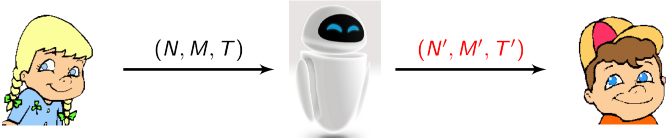

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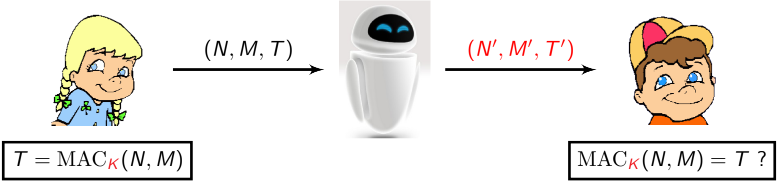

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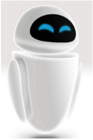

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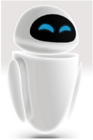

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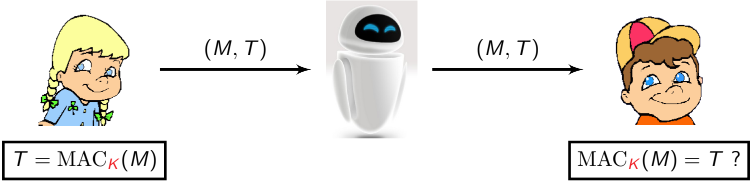

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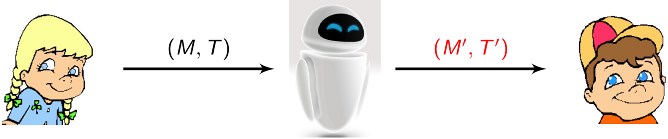

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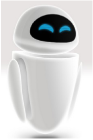

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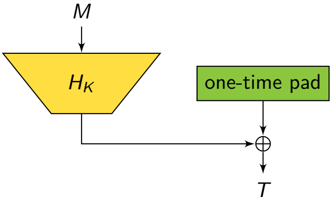
Wegman-Carter MACs [GMS74, WC81]



- based on an ε-almost xor-universal (ε-AXU) hash function H:

$$\forall M \neq M', \forall Y, \Pr[K \leftarrow_\$ \mathcal{K} : H_K(M) \oplus H_K(M') = Y] \leq \varepsilon$$

- in practice, OTPs are replaced by a PRF applied to a nonce N
- H usually based on polynomial evaluation (GCM, Poly1305)
- "optimal" security:

$$\mathbf{Adv}_{\mathrm{WC}}^{\mathrm{MAC}}(q_m, q_v) \leq \varepsilon q_v + \mathbf{Adv}_F^{\mathrm{PRF}}(q_m + q_v)$$

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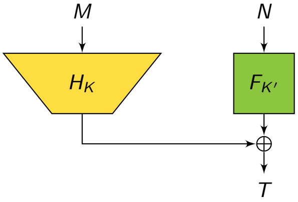

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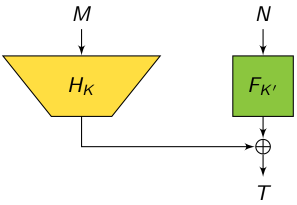

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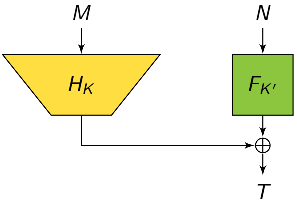

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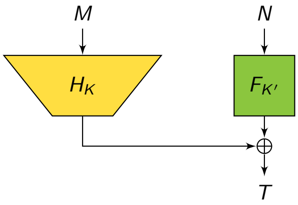

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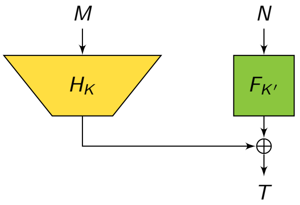

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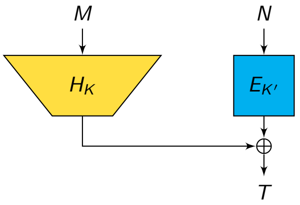

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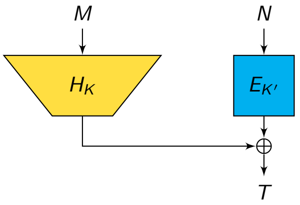

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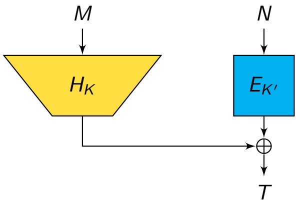

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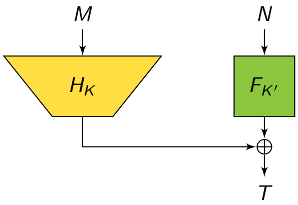

- Wegman-Carter MACs are brittle: a single nonce repetition can completely break security [Jou06, HP08]
- esp. for polynomial-based hashing, i.e., $H_K(M) = P_M(K)$:

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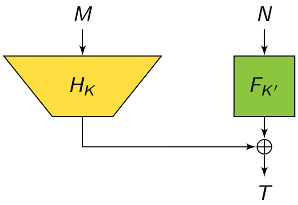

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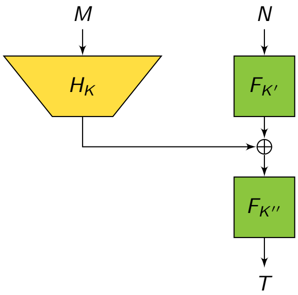

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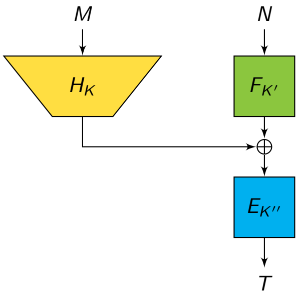

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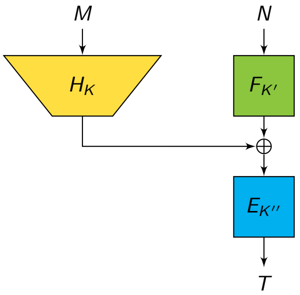

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- BUT security drops to the birthday bound when a nonce is used twice ;
- same problem when implementing F from a Block Cipher ;
- too simple mixing of the nonce and the hash of the message...

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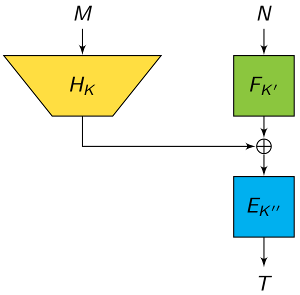

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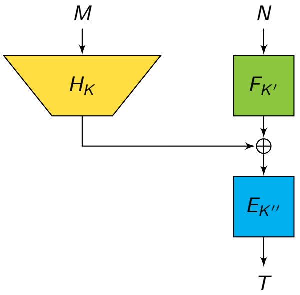

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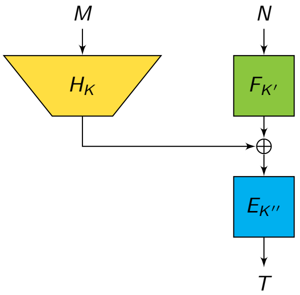

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A Useful Remark

- Take any two sets S, T and a set of triples $\tau = \{(s, t_1, t_1'), \dots, (s, t_q, t_q')\}$ such that

$$\forall 1 \leq i \neq j \leq q,\ s_i = s_j \implies t_i \neq t_j \text{ and } t_i' \neq t_j'.$$

- Take an additional triple $(s, t, t') \notin \tau$.
- Then, the probability that a uniformly random family of permutations $(P_s)_{s \in S} \in \mathrm{Perm}\{T\}^S$ satisfies

$$\forall 1 \leq i \leq q,\ P_{s_i}(t_i) = t_i', \qquad P_s(t) = t'$$

is greater than

$$\left(1 - \frac{1}{2^n - \max(q_1, \dots, q_r)}\right) \prod_{i=1}^{r} \frac{(2^n - q_i)!}{(2^n)!} \qquad (1)$$

where r is the number distinct $s \in S$ in τ, and q_i is the number of occurences of tweak s_i.

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It is possible to build secure MACs using previous remark by ensuring that r remains low !

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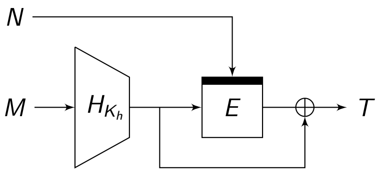

- Dubbed the HENK construction (*Hash-then-Encrypt with Nonce as Key*).
- Based on a BC E and a ε-AXU and uniform hash function H.
- Efficient: 1 call to the BC and 1 call to H.
- Secure: probability of forgery for a (μ, q_m, q_e, q_v)−adversary is lower than

$$\frac{(\mu-1)q_m}{2^n}+(\mu-1)\varepsilon q_m+\frac{q_v}{2^n-\mu-q_e}+(3\mu+n)\varepsilon q_v+\frac{q_e}{2^n-q_e}.$$

- Proof in the Ideal Cipher Model.

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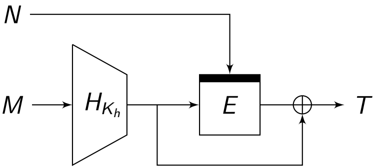

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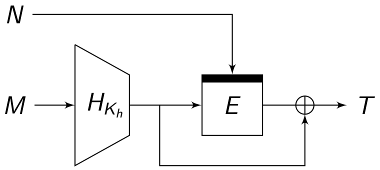
- Proof in the Ideal Cipher Model.

A Nonce-Based MAC



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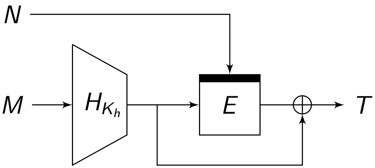

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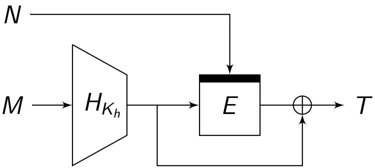

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- Dubbed the HERK construction (*Hash-then-Encrypt with Random Key*).
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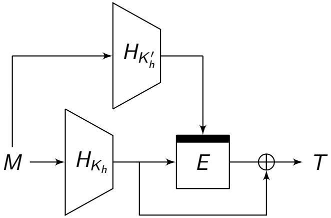
A Standard MAC



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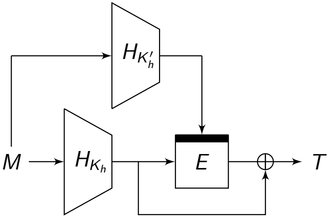

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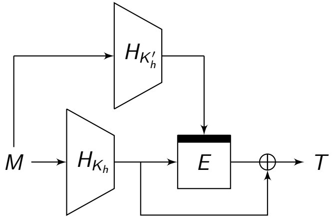

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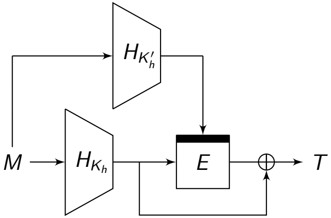

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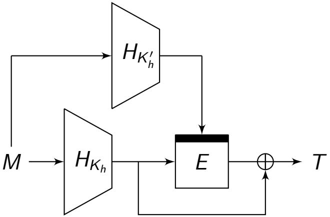

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Outline

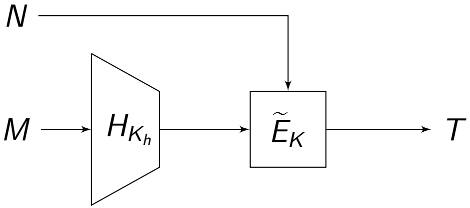
Context

Block Cipher Based Constructions

Tweakable Block Cipher Based Constructions

Security of Truncated MACs

A Nonce-Based MAC



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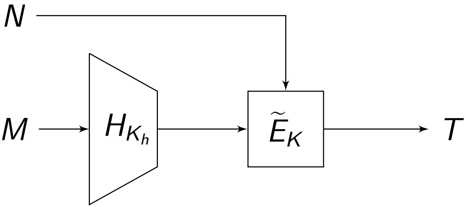

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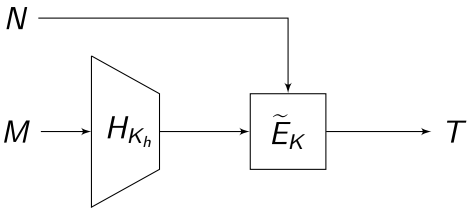

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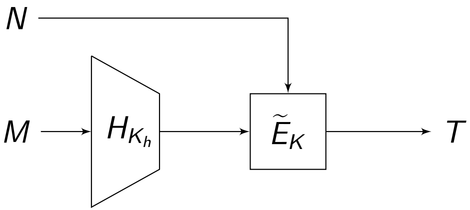

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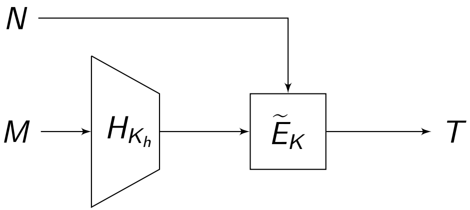

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$$\begin{cases} N_i = N'_j \\ T_i = T'_j \\ H_{K_h}(M_i) = H_{K_h}(M'_j), \end{cases}$$

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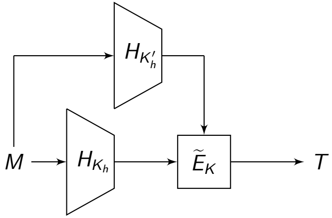

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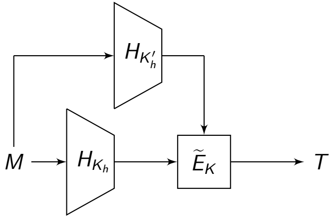

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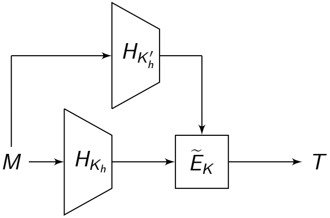

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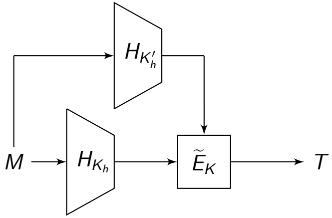

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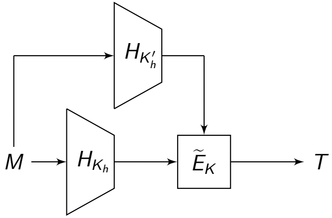

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A Standard MAC (proof)

Before applying Eq 1, we need to make sure that none of the following holds:

- there exist a MAC query $(M, T) \in \tau_m$ and a verification query $(M', T', b) \in \tau_v$ such that $H_{K'_h}(M) = H_{K'_h}(M')$ and $H_{K_h}(M) = H_{K_h}(M')$ and $T = T'$,
- there exists two distinct MAC queries (M, T) and (M', T') such that $H_{K'_h}(M) = H_{K'_h}(M')$ and either $H_{K_h}(M) = H_{K_h}(M')$.

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Outline

Context

Block Cipher Based Constructions

Tweakable Block Cipher Based Constructions

Security of Truncated MACs

What about truncated variations of our constructions ?

Our construction compose well with truncation.

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Our construction compose well with truncation.
E.g., if one takes the first s bits of the outputs of the HEHT construction, the probability of forgery of a $(q_m, q_v)-$adversary is lower than

$$\mathbf{Adv}_{\widetilde{E}}^{\mathrm{TPRP}}(\mathsf{A}') + 2\varepsilon^2 q_m^2 + 2^{n-s}\varepsilon^2 q_m q_v + \frac{2^{n-s} q_v}{2^n - q_m}.$$

The end...

Thanks for your attention !

Any questions ?

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